Teacher: Panagiotis PAPADOPOULOS
Subject: Mathematics

Title: Solving Quadratic Equations using the Quadratic Formula	**Time: 45 minutes**
Subject: *Mathematics*	
Aims: • To develop students' understanding and application of the quadratic formula to solve quadratic equations. • To foster problem-solving skills by using computational thinking principles such as decomposition, pattern recognition, abstraction, and algorithm design to approach and solve quadratic equations systematically.	
Key CS elements: Decomposition; Pattern recognition; Abstraction; Algorithm design.	
Age group: 14-16 years old	
Learning situations: classroom, IT lab	**Activity type:** analysis
Resources: **1. Whiteboard and markers** **2. Quadratic formula handouts** **3. Graphing calculators or computers (optional)** **4. Geogebra (geogebra.org)**	
Learning development:	

Problem definition:

Introduction (10 minutes)

- Briefly introduce the concept of quadratic equations ($ax^2 + bx + c = 0$).
- Discuss how the quadratic formula is used to find solutions (roots) for these equations.
  $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$

1. **Decomposition (10 minutes)**
   - **Explain Decomposition**: Break down the quadratic formula into smaller, manageable parts for the students to understand:
     1. Identify coefficients: a, b, and c from the equation.
     2. Calculate the **discriminant**: b^2-4ac
     3. Find the square root of the discriminant.

   Activity: Ask students to decompose a given quadratic equation and identify the values of a, b, and c. Use the positive and negative versions of the formula to solve for x.

2. **Pattern Recognition (5 minutes)**
   - **Explain Pattern Recognition**: Identify common characteristics or patterns in the discriminant:
     - If the discriminant is positive, there are two real roots.
     - If the discriminant is zero, there is one real root (double root).
     - If the discriminant is negative, there are no real roots (complex roots).

   Activity: Provide several quadratic equations and have students determine the number of roots by examining the discriminant.

3. **Abstraction (10 minutes)**

   - **Explain Abstraction**: Abstract the concept of solving quadratic equations by focusing only on the essential elements: the quadratic formula, its application, and interpreting results.

Activity: Give students various quadratic equations. Ask them to abstractly represent the key steps needed to solve any quadratic equation using the formula, without focusing on specifics like large numbers or complex solutions.

4. Algorithm design (10 minutes)

- **Explain Algorithm Design**: Develop a step-by-step process to solve quadratic equations using the quadratic formula

Algorithm for Solving a Quadratic Equation:

1. **Step 1: Identify the coefficients**
   - Extract the coefficients a, b, and c from the quadratic equation $ax^2 + bx + c = 0$.
2. **Step 2: Compute the discriminant**
   - Use the formula for the discriminant: $\Delta = b^2 - 4ac$.
3. **Step 3: Check the discriminant**
   - If $\Delta > 0$: There are two distinct real solutions.
   - If $\Delta = 0$: There is one real solution (a double root).
   - If $\Delta < 0$: There are no real solutions, but there are two complex solutions.
4. **Step 4: Solve for real or complex solutions**
   - For $\Delta \geq 0$: Use the quadratic formula to find the real solutions.

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

   - For $\Delta < 0$: Calculate the imaginary parts of the solutions.
     - Extract the square root of the negative discriminant $\sqrt{-\Delta}$.
     - The solutions will be of the form:

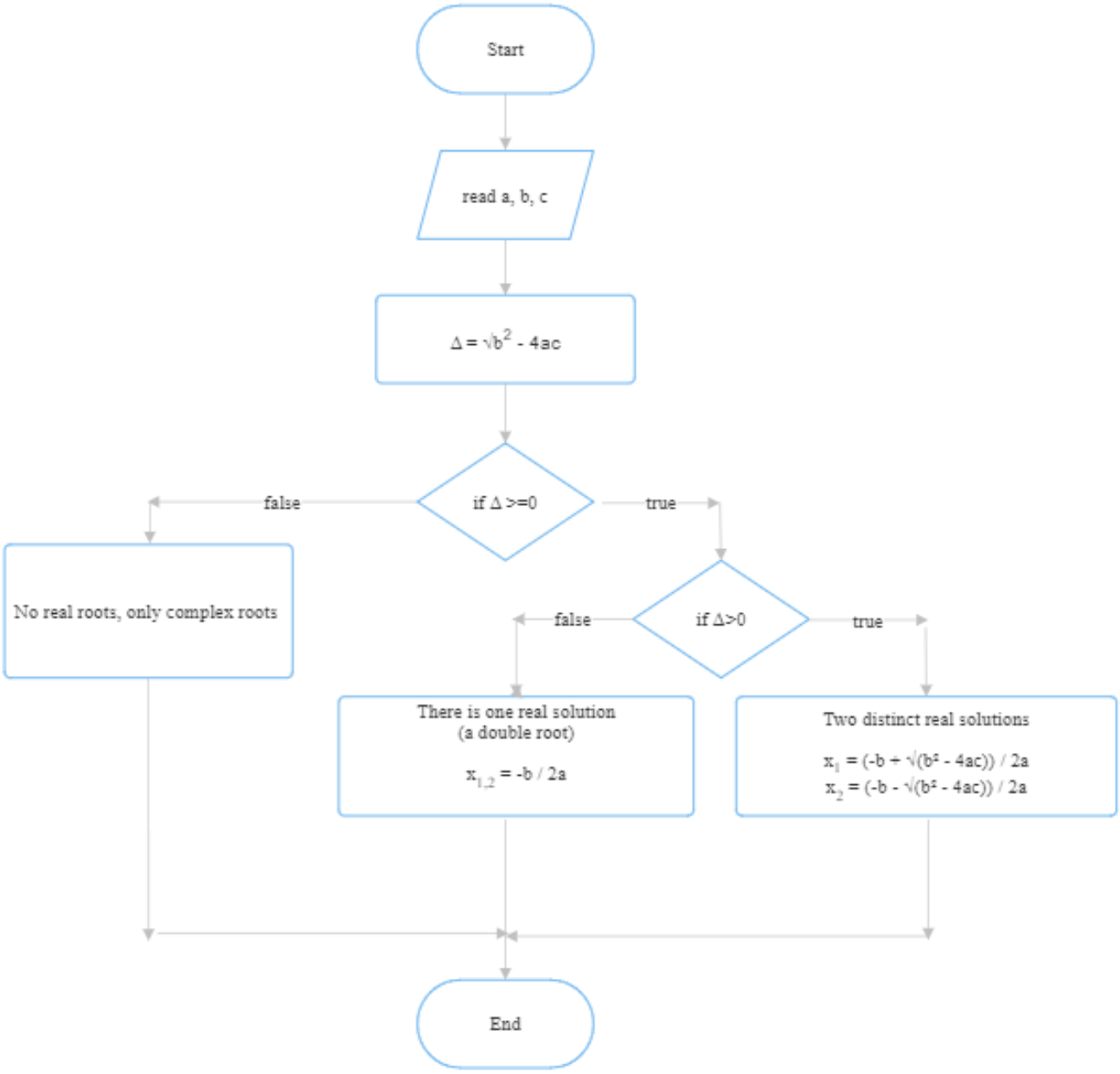
$$x = \frac{-b}{2a} \pm \frac{i\sqrt{-\Delta}}{2a}$$

       where i is the imaginary unit.
5. **Step 5: Interpret the solutions**
   - If the solutions are real, interpret them in the context of the problem.
   - If the solutions are complex, explain that the equation has no real roots but has complex solutions.

Note: Case Δ < 0 in Step4 is only for pupils familiar with complex numbers

Activity: Have students solve a quadratic equation step-by-step, following the algorithm they designed.

Flowchart of the quadratic formula



Assessment Test (20 minutes):

Assessment Test on Solving Quadratic Equations using the Quadratic Formula
Time: 20 minutes
Instructions: Answer the following questions. Show all your work for full credit.

Part 1: Conceptual Understanding (5 points each)

1. What is the quadratic formula? Write it down.
2. What is the discriminant of a quadratic equation, and how does it help in determining the nature of the roots?
3. Explain what happens when:

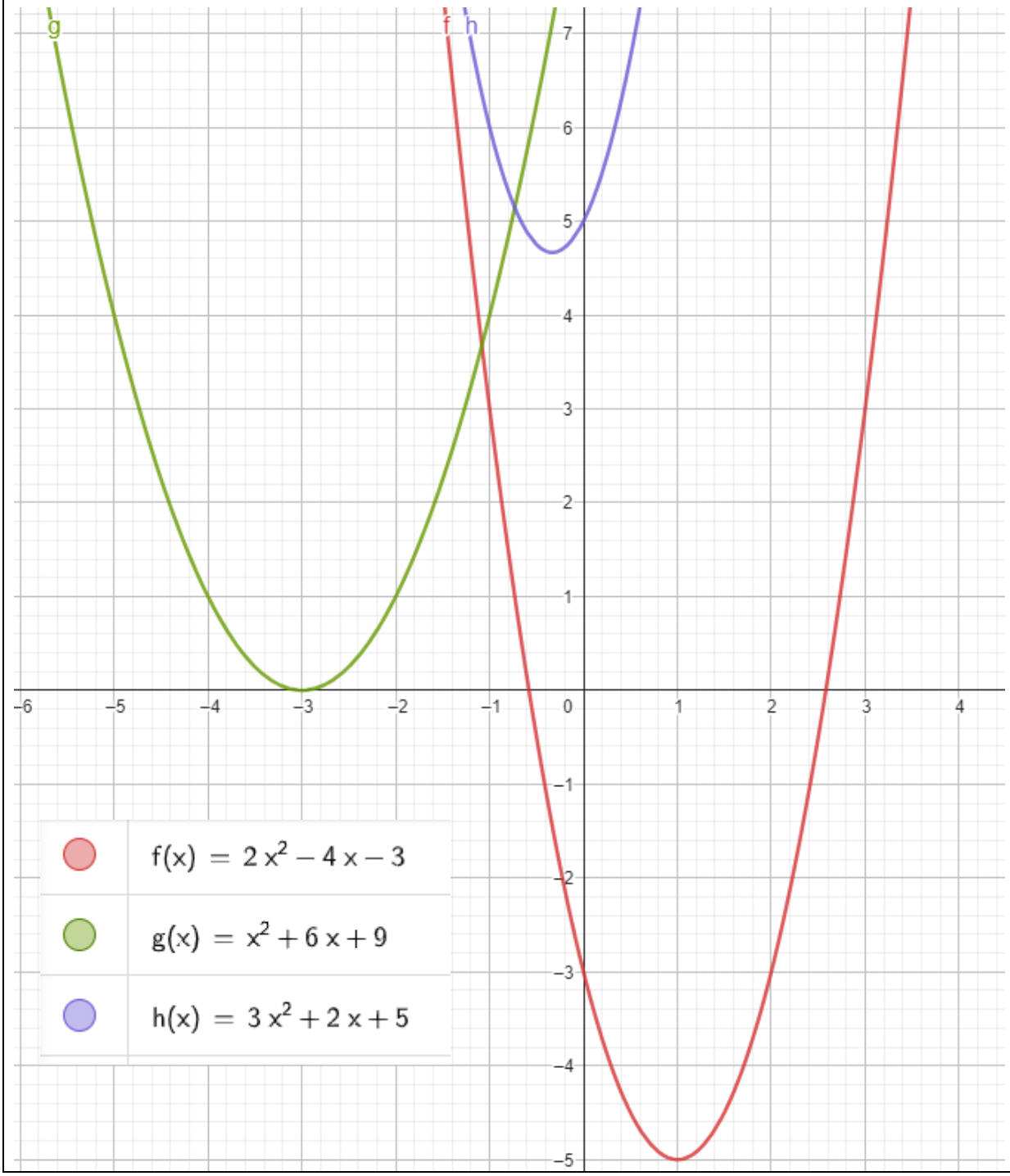
- The discriminant is greater than 0.
- The discriminant is equal to 0.
- The discriminant is less than 0.

Part 2: Solving Quadratic Equations (10 points each)

Solve the following quadratic equations using the quadratic formula. Be sure to calculate the discriminant first and explain the nature of the roots.

4. $2x^2 - 4x - 3 = 0$
5. $x^2 + 6x + 9 = 0$
6. $3x^2 + 2x + 5 = 0$

Graphs of above functions



Part 3: Word Problem (10 points)

7. A ball is thrown upward with an initial velocity of 15 m/s from a height of 25 meters. The heigh h of the ball at any time ttt is given by the equation: $h(t)=-5t^2+15t+25$
   Use the quadratic formula to find out when the ball will hit the ground (when $h(t)=0$).

Bonus Question (5 points)

8. Explain why a quadratic equation might not have any real solutions. Provide an example.

Expected results:

- Students will be able to solve quadratic equations using the quadratic formula.
- They will understand how to break down complex problems, recognize patterns, and create step-by-step solutions in a logical manner.

Note: